

$b \rightarrow s\gamma$ and CP violation in the MSSM

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In this work we study possible new contributions to ε_K and ε_B in the MSSM with large supersymmetric phases. We show that, in the CMSSM, the constraints coming from the experimental measure of the $b \rightarrow s\gamma$ decay imply that these contributions are too small to be detected in CP violation experiments with the available sensitivity.

The minimal supersymmetric extension of the SM (MSSM) contains new observable phases which can cause deviations from the predictions of the SM in CP violation experiments. In fact, in the so-called Constrained Minimal Supersymmetric Standard Model (CMSSM) with strict universality at the GUT scale there are two new phases present. These phases can be chosen to be the phases of the μ parameter (φ_μ) and the trilinear soft coupling, (φ_{A_0}).

It is well-known that for most of the CMSSM parameter space the experimental bounds on the electric dipole moments of the electron and neutron constrain $\varphi_{A_0,\mu}$ to be at most $\mathcal{O}(10^{-2})$. So, these new supersymmetric phases have been taken to vanish exactly in most studies of CMSSM. However, in the last few years the possibility of having non-zero SUSY phases has again attracted a great deal of attention. Several new mechanisms have been proposed to suppress EDMs below the experimental bounds while allowing SUSY phases $\mathcal{O}(1)$ [1].

In this work we are going to study new effects on CP-violation observables in the CMSSM with large supersymmetric phases.

1. Flavor change in the CMSSM

The CMSSM is completely defined at the electroweak scale in terms of $\tan\beta$, the scalar mass m_0^2 , the gaugino mass $m_{1/2}$, the trilinear coupling A_0 and the two phases, φ_A , φ_μ when we require radiative symmetry breaking [2]. Even in this simple model with strict universality, due to the existence of two Yukawa matrices non si-

multaneously diagonalizable, some flavor mixing leaks through RGE into the sfermion mass matrices. In fact, in the SCKM basis, any off-diagonal entry in the sfermion mass matrices at M_W will be necessarily proportional to a product of Yukawa couplings. In the up (down) squark mass matrix the up (down) Yukawas will mainly contribute to diagonal entries while off-diagonal entries will be due to the down (up) Yukawa matrix. This means, for instance, that in this model the off-diagonality in the $m_{LL}^{(d)2}$ matrix will roughly be $c \cdot Y_u Y_u^\dagger$. Where c is a proportionality factor that, in order of magnitude, is roughly $c \simeq 1/(4\pi)^2 \log(M_{Gut}/M_W) \simeq 0.20$ as expected from the loop factor and the running from M_{GUT} to M_W . On the other hand, it is also clear that these flavor changing entries in the down squark mass matrix will be very stable with $\tan\beta$, as for $\tan\beta \gtrsim 1$ the up Yukawa matrix is approximately the same for any $\tan\beta$. For the same reasons, the $\tan\beta$ dependence is very strong in the up squark mass matrix because the down Yukawa matrix grows linearly with $\tan\beta$ for large $\tan\beta$. All these are well-known facts in the different studies of FCNC processes in the framework of the CMSSM [2] and imply that Flavor mixing is still dominantly given by the usual CKM mixing matrix in W-boson, charged Higgs and chargino vertices.

In this analysis we are specially interested on CP violating observables, and then we must also consider the presence of observable phases in the sfermion mass matrices. In the following we take $\delta_{CKM} = 0$ to isolate pure effects of the new super-

symmetric phases on CP violating observables. Then, before RGE evolution the susy phases (φ_A , φ_μ) are confined to the left-right part of the sfermion mass matrix while both the left-left, m_Q^2 , and right-right, $m_{D,U}^2$, are real diagonal matrices. However this is not strictly true anymore at M_W : φ_A leaks into the off-diagonal elements of these hermitian matrices through RGE evolution. From the explicit RGE in the MSSM [2], it is clear that this phase only enters the $(m_Q^2)_{ij}$ evolution through the combinations $(A_U A_U^\dagger)_{ij}$ or $(A_D A_D^\dagger)_{ij}$. At M_{GUT} these matrices have a common phase and so the combination (AA^\dagger) is exactly real. So, to the extent that the A matrices keep a uniform phase during RGE evolution no phase will leak into the m_{LL}^2 matrices. However, this is not the case and different elements of the A matrices are renormalized differently. At M_W the general form of this matrix in terms of the initial conditions is,

$$m_Q^2(M_Z) = \eta_Q^{(m)} m_0^2 + \eta_Q^{(A)} A_0^2 + \eta_Q^{(g)} m_{1/2}^2 \quad (1)$$

$$+ \left(\eta_Q^{(gA)} e^{i\varphi_A} + \eta_Q^{(gA)T} e^{-i\varphi_A} \right) A_0 m_{1/2}$$

where the coefficients η are 3×3 matrices with real numerical entries. In this expression we can see that the presence of imaginary parts will be linked to the non-symmetric part of the $\eta_Q^{(gA)}$ matrices. We have explicitly checked that these non-symmetric parts of m_Q^2 are present only in one part per $2 - 3 \times 10^3$ [4]. This situation was also found by Bertolini and Vissani in the CMSSM without new phases for the leakage of δ_{CKM} [5]. So, we conclude that in the processes we will consider we can take the $m_{LL}^{(u)}$ and $m_{LL}^{(d)}$ as real to a very good approximation.

2. Indirect CP violation in the CMSSM

In the SM neutral meson mixing arises at one loop through the well-known W -box. However, in the CMSSM there are new contributions to $\Delta F = 2$ processes coming from boxes mediated by supersymmetric particles. These are, charged Higgs boxes (H^\pm), chargino boxes (χ^\pm) and gluino-neutralino boxes (\tilde{g}, χ^0). The amount of the indirect CP violation in the neutral meson

\mathcal{M} system is measured by the well-known $\varepsilon_{\mathcal{M}}$ parameter, and it depends on the matrix elements of the $\Delta F = 2$ Hamiltonian, $\mathcal{H}_{eff}^{\Delta F=2}$,

$$\mathcal{H}_{eff}^{\Delta F=2} = -\frac{G_F^2 M_W^2}{(2\pi)^2} (V_{td}^* V_{tq})^2 (C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu) + C_3(\mu) Q_3(\mu)) \quad (2)$$

where the operators are $Q_1 = (\bar{d}_L^\alpha \gamma^\mu q_L^\alpha)^2$, $Q_2 = (\bar{d}_L^\alpha q_R^\alpha)^2$ and $Q_3 = \bar{d}_L^\alpha q_R^\beta \cdot \bar{d}_L^\beta q_R^\alpha$. Here, $q = s, b$ for the K and B -systems respectively and α, β are color indices. These are the only three operators present in the limit of vanishing m_d .

The three operators in Eq (2) are very different with respect to the flavor mixing in the sfermion mass matrices. The operator Q_1 preserves chirality along the fermionic line while the operators Q_2 and Q_3 change chirality. This means, for instance, that gluino contributions to C_1 will not need a chirality change in the sfermion propagator and hence will involve mainly the $m_{LL}^{(d)}$ submatrix, real to a very good approximation. The operators Q_2 and Q_3 always involve a change in the chirality of the external quarks and consequently also a change of the chirality of the associated squarks or gauginos. This implies the presence of the new supersymmetric phases.

In first place we will consider the Wilson coefficient C_1 . All the different supersymmetric boxes contribute to this operator. Both the usual SM W -box and the charged Higgs box do contribute to C_1 , however, in the absence of CKM phase, they can only contribute to the mass difference $\Delta M_{\mathcal{M}}$, but never to the imaginary part in $\varepsilon_{\mathcal{M}}$. In the gluino contribution the source of flavor mixing is not directly the usual CKM matrix, but it is the presence of off-diagonal elements in the sfermion mass matrices discussed in the previous section. Working in the SCKM basis all gluino vertices are flavor diagonal and real. This means that in the MI approximation we need a complex mass insertion in one of the sfermion lines. We have seen in the previous section that these mass insertions are real in one part per 2×10^{-3} . The values for the real and imaginary parts of the mass insertions required to saturate $\Delta M_{\mathcal{M}}$ and ε_K are, $\sqrt{|Re(\delta_{12}^d)_{LL}^2|} < 4 \cdot 10^{-2}$ and $|\delta_{12}^d)_{LL}| \sin(2\phi_{LL}) < 3 \cdot 10^{-3}$. Taking into ac-

count the relation found in the previous section between real and imaginary parts, respecting the bound on the real part implies that no sizeable contributions to ε_K can be found. The situation in $B^0-\bar{B}^0$ mixing is completely analogous.

The chargino can also contribute to C_1 . In this case flavor mixing comes explicitly from the CKM mixing matrix and flavor mixing in the sfermion mass matrices plays only a secondary role. In the approximation of no intergenerational mixing in the sfermion mass matrices we have [3],

$$C_1^\chi(\mu_0) = \frac{1}{4} \sum_{i,j=1}^2 \sum_{k,l=1}^6 \sum_{\alpha\beta} \frac{V_{\alpha d}^* V_{\alpha q} V_{\beta d}^* V_{\beta q}}{(V_{td}^* V_{tq})^2} \quad (3)$$

$$\left(G^{(\alpha,k)i} G^{(\alpha,k)j*} G^{(\beta,l)i*} G^{(\beta,l)j} Y_1(z_k, z_l, s_i, s_j) \right)$$

where $z_k = M_{u_k}^2/M_W^2$, $s_i = M_{\chi_i}^2/M_W^2$, and $G^{(\alpha,k)i}$ represent the coupling of chargino and squark k to left-handed down quarks. All these couplings and the loop function $Y_1(a, b, c, d)$ can be found in [3]. From Eq.(3), taking into account that $Y_1(a, b, c, d)$ is symmetric under the exchange of any pair of arguments we can easily see that this contribution is exactly real. We have explicitly checked that the presence of intergenerational mixing in the sfermion mass matrices does not modify this fact at an observable level. Imaginary parts appear at least two orders of magnitude below the corresponding real parts. Hence we will not consider them here, more details will be given in a complete paper [4].

Now we analyze the contributions to the C_2 and C_3 Wilson coefficients. The charged Higgs box contributes only to Q_2 , but once again the absence of phases prevents it to contribute to $\varepsilon_{\mathcal{M}}$. Gluino boxes contribute both to Q_2 and Q_3 , however flavor change will be given in this case by a left-right mass insertion that in the CMSSM is always proportional to the mass of the right handed squark. This mass insertions have phases $\mathcal{O}(1)$ but the mass suppression will not be compensated in any case by a large value of $\tan\beta$. This means that gluino contributions will always be smaller than the corresponding chargino ones.

The most important contribution will usually be the chargino box. Before the inclusion of QCD effects it contributes solely to the coefficient C_3 .

At first approximation CKM produces directly all the necessary flavor change, then we have,

$$C_3^\chi(M_W) = \sum_{i,j=1}^2 \sum_{k,l=1}^2 [F_s(3, k, 3, l, i, j) - \quad (4)$$

$$2F_s(3, k, 1, l, i, j) + F_s(1, k, 1, l, i, j)]$$

with the functions $F_s(\alpha, k, \beta, l, i, j) = H^{q(\alpha,k)i} G^{(\alpha,k)j*} G^{(\beta,l)i*} H^{q(\beta,l)j} Y_2(z_{\alpha k}, z_{\beta l}, s_i, s_j)$ and $H^{q(\alpha,k)i}$ the coupling of chargino and squark to the right-handed down quark q [3]. We have used CKM unitarity and degeneracy of the first two generation squarks. In this case, due to the differences between the H and G couplings this contribution is always complex in the presence of susy phases. Recently we showed in a short letter [6], that this contribution could be relevant for $\varepsilon_{\mathcal{M}}$ in the large $\tan\beta$ regime. However, in that work we did not take into account the additional constraints coming from $b \rightarrow s\gamma$ decay. In the next sections we will analyze the relation of $\varepsilon_{\mathcal{M}}$ with this decay, and the constraints imposed by the experimental measure.

3. $b \rightarrow s\gamma$ decay

The decay $b \rightarrow s\gamma$ has already been extensively studied in the context of the CMSSM with vanishing susy phases [7]. Being the branching ratio a CP conserving observable, the presence of new phases will not modify the main features found in [7]. However we will see that the experimental measure will also have a large impact on the imaginary parts of the decay amplitude. This decay is described by the following $\Delta F = 1$ effective Hamiltonian

$$\mathcal{H}_{eff}^{\Delta F=1} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=2,7,8} C_i \mathcal{O}_i \quad (5)$$

where the different operators involved are $\mathcal{O}_7 = em_b/(4\pi)^2 \bar{s}_L \sigma^{\mu\nu} F_{\mu\nu} b_R$ that contributes directly to the decay and $\mathcal{O}_2 = \bar{s}_L \gamma_\mu c_L \bar{c}_L \gamma^\mu b_L$ and $\mathcal{O}_8 = g_s m_b/(4\pi)^2 \bar{s}_L \sigma^{\mu\nu} G_{\mu\nu} b_R$, that contribute through RGE running. Here $C_2(M_W) = 1$, and the photonic and gluonic penguins receive contributions from W^\pm boson, charged Higgs H^\pm , chargino χ^\pm , neutralino χ^0 , and gluino \tilde{g} loops.

Among these contributions, the W penguin is exactly the same as in the SM and does not depend on any supersymmetric parameters. Similarly, the charged Higgs penguins only depend on m_{h^\pm} from the new susy parameters. The expressions for $\mathcal{C}_7^{W,H}(M_W)$ and $\mathcal{C}_8^{W,H}(M_W)$ can be found for instance in [2,7].

Then, we have also diagrams mediated by neutralinos and gluinos. Flavor change in this diagrams is due to the off-diagonality in the sdown mass matrix. However, this flavor change is always smaller than direct CKM mixing and being a left-right off-diagonal mass insertion is suppressed by m_b . Indeed, smallness of the neutralino and gluino contributions has already been established in [7] where it was shown that, in the CMSSM, such contributions are roughly one order of magnitude smaller than the chargino contribution.

The most important supersymmetric contribution will be in a large part of the parameter space the chargino contribution. This is due to the fact that in this diagram, the chirality change can be made both through a chargino mass insertion in the loop or an external leg mass insertion proportional to m_b . We are then mainly interested in the chargino mass insertion. In terms of the chargino-quark-squark couplings used in the previous section, these contributions are,

$$\mathcal{C}_{7,8}^{\chi^\pm}(M_W) = \sum_{k=1}^2 \sum_{i=1}^2 \frac{m_{\chi^i}}{m_b} (H^{b(3,k)i} G^{*(3,k)i} F_R^{7,8}(z_{3k}, s_i) - H^{b(1,k)i} G^{*(1,k)i} F_R^{7,8}(z_{1k}, s_i)) \quad (6)$$

where the loop functions are defined by $F_R^7(x, y) = [F_1(y/x) + e_U F_2(y/x)]/x$ and $F_R^8(x, y) = F_4(y/x)/x$ with the functions $F_i(x)$ in [2]. Once more we use CKM unitarity and degeneracy of the first two generations of squarks. We can see in this expression that the enhancement due to m_{χ^i}/m_b is partially compensated by the presence of the b yukawa coupling in $H^{b(\alpha,k)i}$, [2]. However this contribution will still be too large for large values of $\tan\beta$.

4. $b \rightarrow s\gamma$ and $\varepsilon_{\mathcal{M}}$ correlated analysis

If we compare Eqs. (4) and (6) we can see that, except the presence of different loop functions and possibly different Yukawa couplings of the down quarks, both chargino contributions to the C_3 and C_7 Wilson coefficients are deeply related. In fact, the couplings $H^{q(\alpha,k)i}$ only depend on the down quark q through its Yukawa coupling $h_q = m_q/(\sqrt{2}M_W \cos\beta)$. If we make a rough approximation and we take all the loop functions in the sum equal for both C_3 and C_7 we would obtain that C_3 is exactly the square of C_7 times m_q^2/M_W^2 . Naturally, this is not at all a good approximation, but we can expect that the order of magnitude of C_3 is determined by the allowed values of C_7 . Following [8], it is possible to constrain in

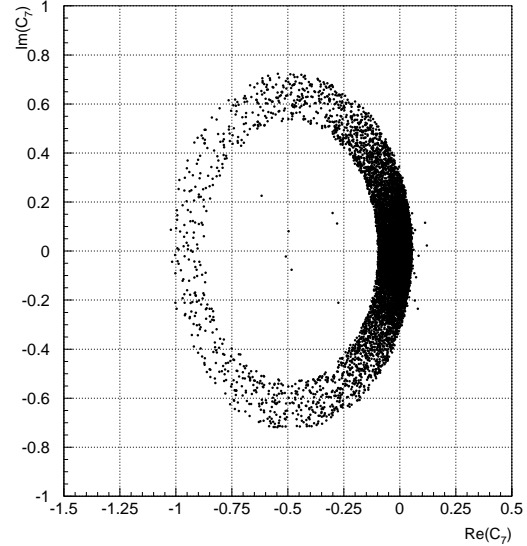


Figure 1. Experimental constraints on the Wilson Coefficient \mathcal{C}_7

a model-independent way new physics contributions to the Wilson coefficients at M_W . In terms

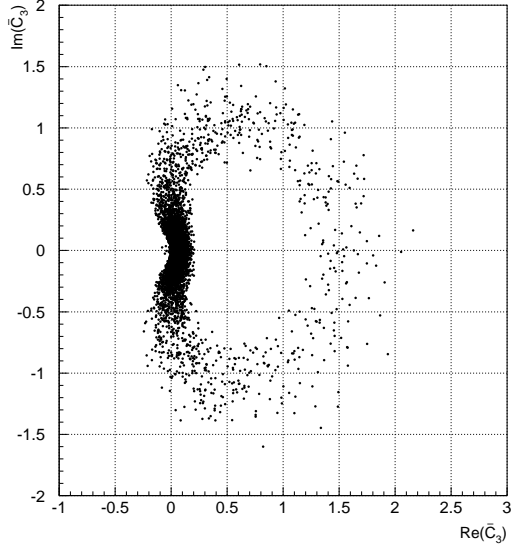


Figure 2. Allowed values for the re-scaled WC \bar{C}_3

of these Wilson coefficients $\text{BR}(B \rightarrow X_s \gamma)$ is,

$$\text{BR}(B \rightarrow X_s \gamma) \simeq 1.17 + 0.38|\xi_7|^2 + 0.015|\xi_8|^2 + 1.39\text{Re}[\xi_7] + 0.156\text{Re}[\xi_8] + 0.083\text{Re}[\xi_7\xi_8^*] \quad (7)$$

where $\xi_a = C_a(M_W)/C_a^{W^\pm}(M_W)$, and the values of the different coefficients are taken from [8]. Then, with the experimental measure, $\text{BR}(B \rightarrow X_s \gamma) = (3.14 \pm 0.48) \times 10^{-4}$, we can constrain the allowed values of the complex variables ξ_7 and ξ_8 . In fact, we can already see from Eq. (7), that in the approximation $\xi_7 \approx \xi_8$, this is simply the equation of an ellipse in the $\text{Re}[\xi_7]$ - $\text{Im}[\xi_7]$ plane. In the case of supersymmetry the new physics contribution to ξ_7 and ξ_8 will be mainly due to the chargino contributions as we have discussed in the previous section. The allowed values of ξ_7 constrain then directly the chargino contributions to $C_7(M_W)$ and indirectly the values of $C_3(M_W)$.

In figure 1 we show a scatter plot of the allowed values of $\text{Re}(\xi_7)$ versus $\text{Im}(\xi_7)$ in the CMSSM for a fixed value of $\tan\beta = 40$ with the constraints from Eq. (7). The fact that now ξ_7 and ξ_8 are independent does not modify strongly

the shape of the plot. This value of $\tan\beta$ could give rise to observable CP violation [6]. However the shape of the plot is clearly independent of $\tan\beta$, only the number of allowed points and its location in the allowed area depend on the value of $\tan\beta$ considered. Figure 2 shows the allowed values for the re-scaled Wilson coefficient $\bar{C}_3(M_W) = M_W^2/M_q^2 C_3(M_W)$ corresponding to the same parameter space points shown in figure 1. As expected the allowed values for \bar{C}_3 are close to the square of the values of C_7 in figure 1 slightly scaled by different values of the loop functions. We can immediately translate this result to a constrain on the size of the chargino contributions to $\varepsilon_{\mathcal{M}}$.

$$\varepsilon_{\mathcal{M}} = \frac{G_F^2 M_W^2 F_{\mathcal{M}}^2 M_{\mathcal{M}} \eta_3 (V_{td} V_{tq})^2 M_{\mathcal{M}}^2}{4\pi^2 \sqrt{2} \Delta M_{\mathcal{M}} 24 m_q^2} \text{Im}[C_3] \quad (8)$$

So, for ε_K we have $C_3 = m_s^2/M_W^2 \bar{C}_3$ and this implies that $\varepsilon_K^\chi \lesssim 0.5 \times 10^{-7}$. Then in this simple model, even with large susy phases, ε_K will be mainly given by the usual SM box. Similarly $\varepsilon_B^\chi \approx 0.4 \times 10^{-3} \text{Im}[\bar{C}_3]$ and hence out of reach in the forthcoming B factories.

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